Received: *September 11, 2000*, Accepted: *November 7, 2000*

Stringy world branes and exponential hierarchies

Peter Mayr

CERN Theory Division, CH-1211 Geneva 23, Switzerland
E-mail: Peter.Mayr@cern.ch

Abstract: We describe heterotic string and M-theory realizations of the Randall-Sundrum (RS) scenario with $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetry in the bulk. Supersymmetry can be broken only on the world brane, a scenario that has been proposed to account for the smallness of the cosmological constant. An interesting prediction from string duality is the generation of a warp factor for conventional type-II Calabi-Yau 3-fold compactifications. On the other hand we argue that an assumption that is needed in the RS explanation of the hierarchy is hard to satisfy in the string theory context.

Keywords: Extra Large Dimensions, Superstrings and Heterotic Strings, M-Theory, String Duality.

JHEP11(2000)013

Contents

1. Introduction

An appealing interpretation of the hierarchy problem is in terms of a world brane scenario with large extra dimensions that dilute gravity [1, 2, 3]. It was further argued in [4], referred to as RS in the following, that the remaining problem of the unnatural large extra dimension can be eased if the back-reaction of gravity to the brane produces a total space-time with suitable non-product structure. Specifically the metric considered in these papers is of the warped form

$$ds^2 = e^{A(y)}\,\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \tag{1.1}$$

where $\mu,\nu = 1,\ldots,4$, $A(y) = -k|y|$ and y takes values in a finite interval. Due to the exponential dependence of the warp factor on the transverse dimension, a moderate distance in the y direction

$$\delta y \sim 2\,\ln\frac{m}{M} \tag{1.2}$$

may account for the large hierarchy $m_{\mathrm{EW}}/M_{\mathrm{pl}}$ between the scales of gravity and electro-weak interactions.

Note that there is an important assumption in this argument, namely that the length scale δy is the input which assumes a natural value whereas the ratio of mass scales M/m is the derived quantity. In the framework of ref. [4], which is supposed to be the effective description of a desired setup, the validity of this assumption can not be decided. If in contrary physics tells us that M/m is the input quantity, naturally of order one, and δy is a *derived* quantity, the logarithmic relation (1.2) is largely irrelevant for the hierarchy problem. We will argue that this is indeed the generic answer one finds in a string theory realization of this geometry. Nevertheless there are also indications that a sufficiently large interval δy may be stabilized in the case of heterotic and M-theory world branes.

An universal warp factor present in any string theory is the dilaton factor that relates the string sigma model metric $g^{(s)}$ and the canonical Einstein metric

$$g^{(E)}_{MN} = e^{-2\alpha\phi} g^{(s)}_{MN} \,, \tag{1.3}$$

where α is a positive number. It follows that a compactification on a manifold X with a dilaton that depends on the internal dimensions gives rise to a warp factor in space-time. If there are degrees of freedom localized on a "world brane", they may probe this warp factor and the relation that links the masses as measured in the brane and bulk metric is $m_{\rm brane} = e^{-\alpha\phi} m_{\rm bulk}$.

In this note we argue that the world brane geometries of the RS type appear naturally in heterotic string and M-theory vacua[1] and study some of their properties. Indeed the generic four-dimensional heterotic string vacua, the so-called (0,2) vacua, come with warp factors [9, 10, 11]. Moreover, on a subset of the moduli space there are non-abelian gauge symmetries localized on a 5-brane and the dilaton warp factor near the brane realizes the RS geometry. So in some sense the condition for having a world brane of RS type is not (much) more special than having non-abelian gauge symmetries, which is granted by standard model physics! A similar situation arises for 5-branes in M-theory, a subclass of which can in fact be considered as the strong coupling description of the heterotic 5-branes via M-theory/heterotic duality [12].

The heterotic and M-theory brane worlds are interesting examples of theories that provide equivalent dual descriptions in terms of either a gravity free world volume theory or on the other hand a string theory in a geometric background [13]–[16]. The latter describes the higher-dimensional theory in which the brane is embedded as a soliton. This dual perspectives have led to an intriguing proposal [17] that could explain the observed smallness of the cosmological constant [18], if supersymmetry is broken only on the brane. We find that the heterotic brane worlds realize this situation in a very natural way.

[1] World brane embeddings in orientifold/F-theory have been discussed in [5, 6, 7], see also [8] for a related work in the effective supergravity.

2. Heterotic SO(32) small instantons

2.1 The warp factor

Let us start with the small instanton in the heterotic SO(32) string [20], which has a representation as a 5-brane soliton in the low energy supergravity theory [21].[2] On its world volume lives an $\mathcal{N} = 1$ supersymmetric SU(2) gauge theory with sixteen matter multiplets in the fundamental representation. In the following we take this gauge theory on the brane as a candidate to describe part of the standard model; more general and realistic world volume theories will be mentioned later on.

If the 5-brane is embedded in flat ten-dimensional space-time, the six-dimensional Planck mass on the brane is infinite. For this reasons we will consider the situation where the transverse dimensions are compactified on a 4-manifold Y. It is important however that the gravity on Y is not of the standard form but highly altered by the presence of the 5-brane.

Specifically is well-known [9, 10, 11] that the generic heterotic string compactification comes with a warp factor which is entirely due to a variation of the dilaton as in (1.3). The warp factor does in general not lead to exotic effects for the low energy theory as long as the latter depends only on the quantities averaged over the internal manifold. A more interesting story arises in the presence of the small instantons. Firstly the small instantons introduce the localized degrees of freedom that probe the warp factor. Secondly they lead to an extreme local behavior of the supergravity fields and especially the warp factor.

In the presence of small instantons, the Bianchi identity for the anti-symmetric tensor field strength becomes

$$dH = -\mathrm{tr} R \wedge R + \frac{1}{30}\mathrm{Tr} F \wedge F + \sum_{i=1}^{N} \hat{\delta}_{5B}\,, \tag{2.1}$$

where $\hat{\delta}$ denotes a 4-form supported on the submanifold on which the N instantons localize. The first two terms on the right-hand side of eq. (2.1) are forced by anomaly cancellation in the ten-dimensional heterotic string [22]. The last term describes the small instanton. It displays the important fact that the latter contribute to the cancellation of gravitational anomalies localized on a 5-brane in space-time [23]. The integral over the right-hand side must vanish for any 4-dimensional compact submanifold Y in space-time:

$$c_2(Y) - c_2(F) - N = 0\,. \tag{2.2}$$

[2]This 5-brane can be visualized as the $(5+1)$-dimensional hypersurface transverse to a 4-plane H with $\int_H F \wedge F = N$ and localized at the center of the instanton in H in the zero size limit.

The equation that determines the local variation of the dilaton — and thus the warp factor — near a generic point in the transverse directions, must be of the same form as that derived in [9] for a transverse K3 space:

$$\Delta e^{2\phi} = -\mathrm{tr} R^2 + \frac{1}{30}\mathrm{Tr} F^2 + \sum_{i=1}^{N} \delta_{5B}\,. \tag{2.3}$$

Locally the solution for the dilaton and the metric will be approximated by the neutral 5-brane solution of ref. [21]. In the string frame one gets for N small instantons located at a point $r = 0$:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{2\phi}\delta_{mn} dy^m dy^n\,, \qquad e^{2\phi} = e^{2\phi_0}\left(1 + \frac{N\alpha'}{r^2}\right), \tag{2.4}$$

where x^μ, $\mu = 1, \ldots, 6$ and y^m, $m = 1, \ldots, 4$ are coordinates in the dimensions parallel and transverse to the 5-brane, respectively. To obtain a four-dimensional gauge theory on the world volume, two tangential directions of the 5-brane will be compactified on a complex curve B with coordinate z. Introducing a new coordinate $y = \ln\, r$, the Einstein metric close to $r = 0$ becomes:

$$ds^2 = c^{-1/2} e^y \eta_{\mu\nu} dx^\mu dx^\nu + c\, dy^2 \ + \ \{g_{z\bar{z}}(B)\ dz\, d\bar{z} \ + c\, d\Omega_3\}\,, \tag{2.5}$$

where $\mu, \nu = 1, \ldots, 4$, $c = e^{2\phi_0} N$ and $d\Omega_n$ is the metric on the n-sphere. Note that the first bracket describes a five-dimensional space with a metric of the RS form (1.1) with the world brane located at $y = -\infty$. The second bracket contains the metric for the additional transverse and compactified parallel dimensions which do not interfere with the warped five-dimensional geometry.

Localization of gravity. Let us compare the global structure of the warp factor as constrained by eq. (2.2) with that in the RS proposals [4]. The first of these paper assumes localization of gravity on a hidden "Planck brane" with a sink in the warp factor at the world brane, whereas the second describes localization of the graviton at a single brane. The behavior of the metric (2.5) of the small instanton is of the first type.

Moreover the non-trivial structure of gravity in the transverse dimensions, namely peaks and sinks for the warp factor corresponding to localization or dilution of the graviton wave functions, is obviously characterized by the non-vanishing of the term $c_2(Y)$. Since it is the integral of the ten-dimensional counter term for gravitational anomalies, it depends on the transverse manifold Y. Specifically, if $c_2(Y) = 0$, then either there is a world-brane and the space has to be non-compact to be consistent with (2.2) — which makes gravity on the six-dimensional brane trivial. Or Y is a flat torus $\mathbb{T}^n$ and the six-dimensional Planck mass is finite, but the exponential sink at the world brane is prohibited by the the absence of appropriate compensating sources on the compact manifold.

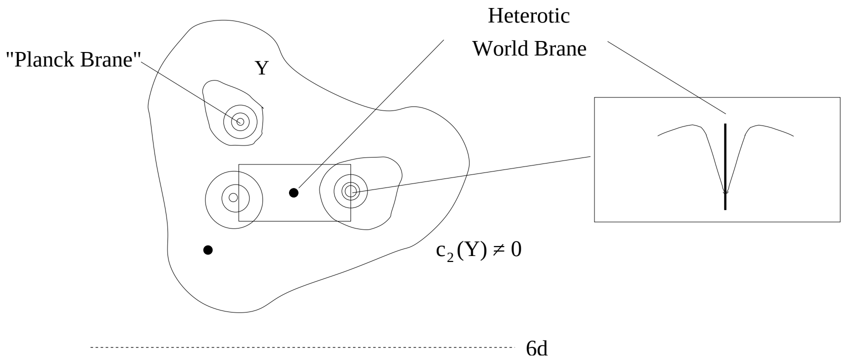

Figure 1: A sketch of the shape of gravity on the manifold Y. The local structure near the small instanton is identical to the RS model, while globally the Planck brane is replaced by sources of the graviton wave function arising from a non-zero value of the "anomaly counter term" $c_2(Y)$.

On the other hand, if Y is a compact space with $c_2(Y) \neq 0$, the global condition (2.2) allows for the presence of 5-branes. The metric is locally determined by (2.5), with an exponential sink at the world brane, and gives a perfect string theory embedding of the RS scenario. The Planck brane is replaced by nearby peaks in the graviton wave-function which are related to the non-vanishing value of the anomaly counter term evaluated on Y. A qualitative picture is shown in figure 1.

Scales and couplings. The simplest case of a transverse compact space Y with $c_2 \neq 0$ is a compact K3 manifold with $c_2 = 24$. To obtain a four-dimensional gauge theory the six-dimensional world volume will be further compactified on a complex curve B. For the moment we assume that B is a $\mathbb{T}^2$ which gives the total manifold a product structure $K3 \times \mathbb{T}^2$. The following discussion of scales and couplings generalizes straightforwardly to the more general compactifications discussed in section 4.

The six-dimensional gravitational coupling constant is given by

$$M_{\rm pl}^4 = M_{\rm str}^8 \int_Y \sqrt{g(Y)} e^{-2\phi} = e^{-2\phi_0} M_{\rm str}^4 , \qquad e^{-2\phi_0} = \frac{M_{\rm str}^4 V_Y}{\lambda_H^2} , \tag{2.6}$$

where λ_H is the ten-dimensional dilaton and V_Y is the volume of K3 without the warp factor in string units. In the second relation we have used the fact that we can calculate the value of the integral for a configuration with constant dilaton. Moreover, in the string frame, the value of the six-dimensional gauge couplings in the perturbative gauge sector and on the brane are [20]:

$$\frac{1}{g_{6,p}^2} \sim M_{\rm str}^2 \, e^{-2\phi_0} , \qquad \frac{1}{g_{6,b}^2} \sim M_{\rm str}^2 . \tag{2.7}$$

Note that, contrary to what happens for the perturbative gauge groups where one has the relation $g_6^2 M_{\rm str}^2 \sim (M_{\rm str}/M_{\rm pl})^4$, the string scale can be arbitrarily small for a fixed gauge coupling and fixed Planck mass by making the volume V_Y large or the coupling λ_H small. This has led to the idea of weak scale superstrings [2, 1].[3] They realize a world localized on a brane with gravity diluted by extra large dimensions, an idea which is also central to refs. [3]. The four-dimensional couplings and scales follow from a trivial multiplication of the six-dimensional quantities with the volume of B, as the world volume fills the entire six-dimensional space time.

However, the above discussion neglects the warp factor. Even if $M_{\rm str} \sim M_{\rm pl}$, that is there is no large dimension, the hierarchy can be generated by an exponential warp factor $\epsilon \equiv e^{-\delta y/2} \sim m_{\rm EW}/M_{\rm pl}$ in a finite interval δy. From eq. (2.4) it follows that the upper end of this interval, which we may choose to correspond to $y = 0$, is set by a multiple of the string scale $M_{\rm str}$. The hierarchy ϵ is then determined by the lower end, or IR cutoff, $y = -\delta y$. We will comment on the question to which extent a large interval $\delta y/2 \sim 20$ is natural and stable in the present situation in section 6. Of course, more generally the hierarchy can also arise from a combined effect of the two mechanisms, with a moderately large dimension leading to an intermediate scale $M_{\rm str} < M_{\rm pl}$ and the remaining hierarchy produced by a warp factor.

3. The cosmological constant and supersymmetry breaking

Irrespectively of the hierarchy problem, there are quite interesting features of the identification of the (heterotic) world brane theories with the observable world. An intriguing property is that a change in the vacuum energy of the world brane fields does not change the effective four-dimensional cosmological constant, differently then what happens, at last naively, for the perturbative, non-localized heterotic gauge sector. Indeed the effective cosmological constant Λ_4 gets contribution from both the brane vacuum energy and the variation of the metric transverse to the brane and its total value is given by an integration constant c:

$$\Lambda_4 = \lambda_{\rm brane} + \lambda_{\rm bulk} + (\partial_y \phi)^2 = c\,, \tag{3.1}$$

where $\lambda_{\rm brane}$ and $\lambda_{\rm bulk}$ are the contributions from the brane and bulk fields other than the metric, respectively. For any chosen value of c, a change in the vacuum energy of the brane is canceled by the appropriate back-reaction of the transverse metric such that Λ_4 is kept fixed. This is the mechanism of Rubakov and Shaposhnikov [24] which was rediscovered in the context of AdS/CFT dualities in refs. [25].

As a consequence, the cosmological constant depends only on the initial value of c, but not on phase transitions or quantum corrections in the gauge theory on the brane. In the present context, a supersymmetric string compactification to a maximal symmetric non-compact space-time M requires M to be Minkowskian [26, 9] and so $c = 0$. In other words, requiring supersymmetry of the *bulk* theory, selects the appropriate boundary condition such that Λ_4 is zero, irrespectively of the vacuum energy of the world brane fields.

[3]The phenomenological viability of the scenario of [2] has been stressed in [19].

The interesting question however is what determines the size of Λ_4 when supersymmetry is broken. If supersymmetry is, in first approximation, only broken on the brane, then the above mechanism still works to cancel the effective cosmological constant from a brane vacuum energy by an adjustment of the transverse metric. A non-zero cosmological constant arises therefore only from the sub-leading effect that the cancellation mechanism will be disturbed by the propagation of supersymmetry breaking into the bulk. Thus the leading order contribution to Λ_4 is determined by the scale $m_{\rm susy}^{\rm bulk}$, rather then $m_{\rm susy}^{\rm brane}$, which characterizes the mass splittings in the observable sector.

Now morally the hierarchy between the mass scales of the bulk and the brane arises from the fact that the interactions between the two type of fields are suppressed by the warp factor $\epsilon = m_{\rm EW}/M_{\rm pl} \sim m_{\rm susy}^{\rm brane}/M_{\rm pl} \ll 1$. The idea of ref. [17] is that if a similar factor suppresses the propagation of supersymmetry breaking back into the bulk, this may lead to a sufficiently small value of Λ_4. In fact it has been argued in ref. [18] that the 1-loop contribution to the cosmological constant induced by the supersymmetry breaking in the bulk leads to values close to the observed data.

To introduce a mechanism that breaks supersymmetry exclusively on the brane is not trivial, however. In the holographic correspondence the mass operator on the brane is dual to the vev of a supergravity bulk field. Generically, an operator that leads to a mass m on the brane corresponds to a mass scale M in the bulk that is *larger*, $m^2 = e^y\, M^2$. It is therefore unclear whether it is even in principle possible to obtain an appropriate warped geometry that describes a supersymmetry breaking sufficiently confined to a neighborhood of the brane in the pure supergravity sector.

An extra sector that comes for free in the case of the heterotic world branes and seems to be tailor-made to realize the above situation is the perturbative gauge sector of the heterotic string with gauge group $G_0 = {\rm SO}(32)$. In an $\mathcal{N}=1$ supersymmetric vacuum, strong gauge interactions in this string sector lead to a fermion condensate at a scale $\Lambda_F \sim e^{-8\pi^2/bg_{4,p}^2}$ that may break supersymmetry non-perturbatively [27]. Let us assume that this scale is very small in the bulk, as a consequence of a small asymptotic value of the dilaton away from the world branes or since G_0 is broken to a subgroup with small b, or a combination of these effects. No matter how small the mass scale Λ_F is away from the five-branes it will grow with decreasing distance and induce a large scale Λ_F of the order of the string scale in the G_0 gauge theory near the brane

$$\Lambda_F(y) \;\sim\; (\Lambda_F)^{\exp(-2\phi)} \;\xrightarrow{\;y\to -\delta y\;}\; (\Lambda_F)^{\epsilon^4} \;\sim\; 1\,, \tag{3.2}$$

in units of $M_{\rm str}$. Supersymmetry breaking is therefore restricted to the immediate local neighborhood of the world branes.

Although the process of supersymmetry breaking is non-perturbative in the heterotic theory and therefore difficult to access, one can, in favorable cases, use

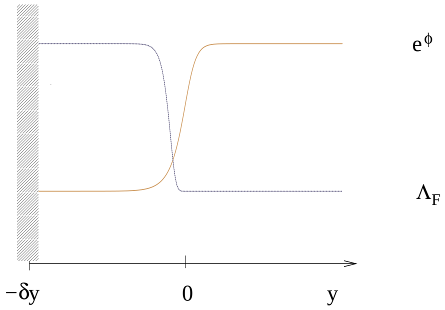

Figure 2: The behavior of the warp factor e^{ϕ} and the mass scale $\Lambda_F(y)$ of the fermion condensate in the perturbative gauge sector as a function of the distance from the brane y. The warp factor decays exponentially towards the brane located to the left. In contrary the gaugino condensate increases to the value $\Lambda_F(-\delta y) \sim M_{\rm str}$ in the neighborhood of the brane.

heterotic/type-II duality to determine these effects by mirror symmetry [28]. We will discuss the heterotic/type-II duality in presence of the world branes in more detail in the next section.

4. Brane compactifications and string duality

We proceed with a short description of four-dimensional compactifications with 5-branes with $\mathcal{N}=2$ and $\mathcal{N}=1$ supersymmetry and discuss some of their properties. An interesting implication arises from heterotic/type-II duality.

4.1 Four-dimensional compactifications with $\mathcal{N}=2$ and $\mathcal{N}=1$ supersymmetry

For simplicity we will mention in the following mainly the SU(2) world volume theory of one small instanton at a generic point in the transverse space-time. Obtaining a phenomenological viable spectrum is, as usual, a question of model building. However it is important to note that the available spectra are sufficiently general to embed the standard model in many ways. Specifically more general world volume theories arise for N coinciding instantons and instantons at non-generic singular points in the transverse space-time. A brief summary of, and references on, these spectra are included in the appendix.

Brane worlds with $\mathcal{N}=2$ supersymmetry. The simplest case of a four-dimensional compactification with 5-branes is the manifold X conformal to K3 times an extra $\mathbb{T}^2$. In addition to the SU(2) world volume gauge theory with matter there is a single hypermultiplet for each 5-brane that parametrizes its position on K3. Moreover, in four-dimensions the gauge theory is $\mathcal{N}=2$ supersymmetric with the extra scalars in the vector multiplets representing the Wilson lines on $\mathbb{T}^2$. There are also Wilson lines in the perturbative SO(32) gauge factor. They parametrize the masses of the fundamental hypermultiplets in the world volume theory.

The compactification on X with non-trivial dilaton in the absence of 5-branes has been studied in [9] and it is entirely straightforward to adapt the discussion to include the 5-branes. As $c_2(X) = 24$, the global constraint (2.2) can be satisfied by taking a trivial gauge background F and adding 24 small instantons transverse to K3. The precise warp factor is determined by (2.3) with the laplacian understood to act only in the four directions of K3. An explicit expression for the curvature terms can be given in the orbifold limit. Including the small instantons, the equation for the warp factor becomes

$$\Delta \frac{e^{2\phi}}{8\pi^2} = -\frac{3}{2}\sum_{k=1}^{16} \delta^4(x - x_k) + \sum_{i=1}^{24} \delta(x - \tilde{x}_i)\,. \tag{4.1}$$

Here x_k are the positions of the 16 orbifold fixed points at which the curvature of K3 has been localized. Moreover the $\tilde{x}_i$ are the locations of the the 24 5-branes.

Brane worlds with perturbative $\mathcal{N} = 1$ supersymmetry. Four-dimensional theories with $\mathcal{N} = 1$ supersymmetry are obtained by compactifying the heterotic string on a Calabi-Yau 3-fold X_3 [26]. Similar as before, evaluating $c_2(S)$ on a holomorphic four-dimensional submanifold S in X_3 determines the possible number of sinks due to world branes transverse to S. The cycle C dual to S is holomorphic, and thus a supersymmetric cycle, on which the 5-brane of the small instanton may be wrapped [1]. For a generic position of the 5-brane the behavior of the local — or low energy — description of the warp factor is still described by (2.4).

The spectrum of the world volume theory of a small instanton wrapped on a general curve C is determined by the normal bundle of the curve.[4] In particular the number of supersymmetries will depend on the dimension of the moduli space of C; an isolated (non movable) cycle C is related to $\mathcal{N} = 1$ supersymmetry while wrapping on curves of genus $g = 1$ may lead to $\mathcal{N} = 2$ supersymmetric theories [30].

4.2 A puzzle from heterotic type-IIA duality

String dualities map the gauge sector of one theory to a gauge sector of in general completely different origin in another theory. It is far from being obvious that they will map a gauge theory on a world brane with exponential warp factor to a localized gauge theory with a similar warp factor in the dual theory.

E.g. the heterotic string has on one hand the perturbative, non-localized gauge theory with gauge group G_0 and on the other hand the gauge theories on the world branes with a warp factor discussed in this paper. For a $K3 \times \mathbb{T}^2$ compactification

[4]See refs. [29] for a discussion of various aspects of such 5-brane wrappings.

this string is dual to a type-IIA compactification on a Calabi-Yau 3-fold X [31], where *both* gauge symmetries arise from localized singularities [32] and thus live on a brane. No warp factor has been found so far in this type-IIA compactification. How can two such theories be dual? The answer to this question is bound to be interesting:

Resolution 1: no warp factor. The first possibility is that the localized (heterotic) world brane theory with a warp factor maps to a theory without a warp factor in the dual (type-IIA) theory. In fact it has been speculated for a while what might be the proper generalization of the AdS/CFT duality to describe a four-dimensional world *with* dynamical gravity, similar to ours. A proper description of the field theory side coupled to gravity must involve string theory, so one expects that AdS/CFT duality is replaced by a suitable string/string duality. In an appropriate scaling limit one must recover string theory on "AdS" on the one side and the "CFT" on the other side. If the type-IIA string metric remains its product form including α' and quantum corrections, it is likely to represent a string duality embedding of a "AdS/CFT" correspondence. In the appropriate limit, the type-IIA string represents the decoupled four-dimensional "CFT" while the heterotic side describes its "AdS" dual.

Resolution 2: a "quantum" warp factor. The second possibility is that the localized theories with warp factor match 1-1 to each other under the duality. In fact we want to argue that this is what happens in the present case of the heterotic/type-IIA duality.

To see which effect in the type-IIA theory might result in a so far neglected warp factor, note that the terms on the right-hand side of eqs. (2.1) and (2.3) are higher order effects in α' in the heterotic string. In fact precisely the same terms have played a crucial role in the equivalence of the hypermultiplet moduli spaces of the heterotic string on K3 singularities of type G and three-dimensional G gauge theories with $\mathcal{N} = 4$ supersymmetry [33]. In particular the α' correction to the heterotic moduli maps to a 1-loop term in the 3d gauge theory. The isomorphicity of these moduli spaces for general G has been established in [34] by showing that the gauge theory comes to life in a type-II compactification[5] which is the string dual of the heterotic string on the K3 singularity times a $\mathbb{T}^3$.

This suggests that a similar effect as the one that corrects the hypermultiplet moduli of the type-IIA theory in the above equivalence may also create a warp factor. An important role must be played by the R^4 terms in the type-II action which contribute to the hypermultiplet metric and the Einstein term in four dimensions [36]. Note that to reproduce the structure of warp factors in the heterotic string, the relevant corrections in the type-II compactification should arise only from those sin-

[5] See also ref. [35] for a M-theory derivation of the equivalence of these moduli spaces.

gularities that describe the non-perturbative world volume theories, but not from the others that support the gauge theories dual to the perturbative gauge group of the heterotic string.

An interesting consequence of such a warp factor for the type-IIA gauge theory would be that we could extend the geometric engineering approach of refs. [30, 37] to describe AdS/CFT duality. In lowest order, the type-IIA vacuum is of the form $X_3 \times M_4$ and describes the (possibly conformal) field theory on flat M_4. Adding the warp factor should deform this geometry locally to one of the type $AdS \times Y_5$, where Y_5 is a 5-dimensional space. Note that there are no RR backgrounds involved and therefore the determination of α' corrections, which is essential for field theories with a low amount of supersymmetry may be available by conventional methods, such as mirror symmetry. More details will appear elsewhere [38].

5. Small E_8 instantons and the M-theory 5-brane

Instead of the small SO(32) instanton, we can also consider the small E_8 instanton which is described by the same equations (2.1)–(2.5). The main difference is the spectrum on the world volume. For a six-dimensional K3 compactification, the SU(2) gauge theory is replaced by a $\mathcal{N} = 1$ tensor multiplet together with the hypermultiplet that describes the location on K3. The $B_{\mu\nu}$ field in the tensor multiplet has an anti-self-dual field strength and couples to a tensionless string [39]. A nice geometric description is in terms of the M-theory dual [12] on $S^1/\mathbb{Z}_2 \times K3$, where the small instanton corresponds to a 5-brane stuck on one of the two end of the world 9-branes. The tensionless strings on the 5-brane represent the boundary of a membrane ending on it. Upon a compactification on a circle we obtain conventional gauge theories. From the tensionless strings one obtains also matter multiplets that are charged under the perturbative gauge symmetry of the heterotic string [6]. In fact a large class of these theories will be equivalent to the theories from the small SO(32) instanton by the usual T-duality [40].

An interesting new branch in the moduli space of these theories is the one where the 5-brane leaves the 9-brane, which corresponds to a vev for the scalar in the tensor multiplet (and a Coulomb branch in the T-dual SO(32) theory). For N coinciding 5-branes in the bulk compactified further on a circle one obtains a SU(N) gauge theory on the world volume theory with one adjoint hypermultiplet [41]. Except for the interactions with the 9-branes, which are suppressed by the distance to them, this 5-brane is the same as the M-theory 5-brane.

Similar as the small heterotic instanton (2.1), the M-theory 5-brane contributes to gravitational anomalies. It also contributes to the Bianchi identity of the 4-form field strength $dF = \sum_k \hat{\delta}_k$. The details of the anomaly cancellations are differ-

[6]See first reference in [29].

ent [42, 43] as the eleven-dimensional supergravity has no anomalies on a smooth manifold. For this reason the 5-branes arise in M-theory compactification with singularities, such as the $S^1/\mathbb{Z}_2$ compactification and the orbifolds without a heterotic dual described in [42, 44]. These singularities contribute the necessary terms in the gravitational equations of motions which take over the role of the term $c_2(X)$ in the case of the perturbative heterotic string. The solution for the warp factor in a neighborhood of N M-theory 5-branes is [45]

$$ds^2 = \triangle^{-1/3}\,\eta_{\mu\nu}dx^\mu dx^\nu + \triangle^{2/3}\,\delta_{mn}dy^m dy^n\,, \qquad \triangle = 1 + \frac{N\,\pi\,l_{11}^3}{r^3}\,. \tag{5.1}$$

After compactification on the curve B this is again of the RS form near the brane location $r = 0$

$$ds^2 = c^{-1/3}\,e^y\,\eta_{\mu\nu}dx^\mu dx^\nu + c^{2/3}\,dy^2 + \left(g_{z\bar z}(B)\ dz\,d\bar z\ + c^{2/3}\,d\Omega_4\right)\,, \tag{5.2}$$

with $c = \pi N$ and $\mu, \nu = 1, \ldots, 4$. The duality between the world volume theory and the supergravity or string background for this case has been discussed in detail in [13, 16].

At this point the reader might wonder why the exponential warp factor did not appear in the five-dimensional supergravity description of refs. [1, 46], which give a *linear* warp factor in the direction separating the 9-branes, usually denoted as x^{11}. However, neither for the heterotic 5-brane which is stuck on the 9-brane, nor the M-theory 5-brane in the bulk, there is a reason to single out the x^{11} direction near the 5-brane. Locally the only natural direction is the radial distance r from the brane in which the warp factor is of the exponential form described in eqs. (2.5) and (5.2).

6. Comments

In this note we argued that world brane geometries of the Randall-Sundrum type appear naturally in generic heterotic string and M-theory compactifications with enhanced non-abelian gauge symmetries. The heterotic and M-theory 5-branes give an interesting and natural example of world branes with a holographic duality and dynamical gravity on the brane. The four-dimensional theories obtained by compactifying two of the brane dimensions are interesting candidates for a "standard model" on the brane. In particular it will be interesting to determine the structure of supersymmetry breaking for $\mathcal{N} = 1$ compactifications and to compare the resulting value of the cosmological constant with the proposals in refs. [17, 18].

On the other hand, although the RS geometry appears naturally in string theory, it does not necessarily lead to an explanation of the hierarchy: why should we consider the length scale δy as more fundamental than the ratio of mass scales m/M? Note that a simple line of argument which would identify the distance of the branes with a modulus ϕ and then assign some natural vev to it applies equally well to the alternative identification of ϕ with the mass and so does not resolve the issue.

More concretely, the UV end, say at $y = 0$, of the interval δy will be set by some multiple of the string scale $c\, M_{\rm str}$. Smaller values of y correspond to the IR flow of the world volume theory. If the world volume theory is conformal, there is an infinitely long homogeneous flow to $y = -\infty$. On the other hand if the theory has a mass scale m, as it should if it serves as a model for a world brane, the exponential flow of the warp factor will stop at $y = 2\, \ln m/M_{\rm str}$. E.g. this is precisely what happens in the case of D3-branes, which have an $\mathcal{N} = 4$ supersymmetric world volume theory [47]. Operators in the world volume theory that break conformal invariance and part of the supersymmetry, such as fermion masses, correspond linearly to vev's of anti-symmetric tensors in the dual supergravity description.

Now the point is that the scale characterizing this anti-symmetric tensor backgrounds, as well as any other generic mass scale m in the string theory, is naturally of the order of $M_{\rm str}$. Thus the length δy of the exponential flow of the warp factor is a *derived* quantity, proportional to $\ln m/M_{\rm str}$. In particular the natural value of the length scale δy is very small in string units and does not account for the required hierarchy.

It should therefore be clear that a situation in string theory, where δy is naturally of order one must be a very peculiar one. In this respect it does not seem that the exponential warp factor leads to a substantial facilitation of the hierarchy problem in the well-defined context of string theory.

However it is worth pointing out that the situation for the heterotic world branes is at least more promising than for D-branes in this respect. The reason is that the mere existence of the world volume degrees of freedom implies that however small the asymptotic string coupling e^{ϕ_0} is away from the brane, the string coupling e^{ϕ} has to be at least of order one at the brane [20]. So the quantity $e^{\delta\phi} = e^{\phi-\phi_0} \sim \epsilon^{-2}$ which characterizes at the same time the hierarchy between the scales on the brane and in the bulk, can be large.[7] It would be interesting to study further whether this property of the world branes considered in this note may be used to make the RS argument work in a string theory context.

Acknowledgments

I would like to thank the CIT-USC center for theoretical physics for hospitality during the course of this work and Wolfgang Lerche, Christof Schmidhuber, Stephan Stieberger and Nick Warner for valuable discussions. I am also grateful to Andreas Brandhuber, Renata Kallosh, Djordje Minic, Hans-Peter Nilles and Yaron Oz for helpful conversations.

[7] A similar conclusion has been reached in [47] for type-II NS 5-brane solutions which are related by S-duality to the perturbed D3-brane configurations.

A. More references on more world volume theories

For N coinciding small SO(32) instantons the world volume theory is a Sp(N) $\mathcal{N}=1$ supersymmetric gauge theory in six dimensions together with a hypermultiplets in the $(32, 2k)$ representation, and an antisymmetric representation of Sp(k) [20]. For N coinciding small E_8 instantons one obtains a world volume theory of N $\mathcal{N}=2$ tensor multiplets and in addition tensionless strings [39]. Upon compactification on a circle the tensor multiplets become vector multiplets and the strings wrapped on S^1 particles. For a certain subset of the moduli space, the theory will be T-dual to the compactification of the SO(32) small instantons.

A large variation of spectra arises if the N instantons are moved to singular points in the transverse K3 [48]. In particular the E_8 instanton may lead to extra gauge degrees of freedom located at the singularity, while the SO(32) instantons have now also tensor multiplets in their spectrum.

Upon compactification on a complex curve B, the spectrum on the world volume depends on the embedding of B in the space-time and in particular on the number of 1-cycles γ_i in $H_1(B)$; reduction of a six-dimensional vector (tensor) on γ_i leads generically to a scalar (vector). The theory on the world volume is twisted [49] and the precise spectrum depends on the normal bundle to B in space-time. See e.g. [30, 50] for calculations of this type in special cases.

References

[1] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

[2] J.D. Lykken, *Weak scale superstrings*, *Phys. Rev.* **D 54** (1996) 3693 [hep-th/9603133].

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[4] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[5] H. Verlinde, *Holography and compactification*, *Nucl. Phys.* **B 580** (2000) 264 [hep-th/9906182].

[6] C.S. Chan, P.L. Paul and H. Verlinde, *A note on warped string compactification*, *Nucl. Phys.* **B 581** (2000) 156 [hep-th/0003236].

JHEP11(2000)013

[7] B.R. Greene, K. Schalm and G. Shiu, *Warped compactifications in m and f theory*, *Nucl. Phys.* **B 584** (2000) 480 [hep-th/0004103].

[8] K. Behrndt and M. Cvetič, *Supersymmetric domain wall world from $D = 5$ simple gauged supergravity*, *Phys. Lett.* **B 475** (2000) 253 [hep-th/9909058].

[9] A. Strominger, *Superstrings with torsion*, *Nucl. Phys.* **B 274** (1986) 253.

[10] B. de Wit, D.J. Smit and N.D. H. Dass, *Residual supersymmetry of compactified $D = 10$ supergravity*, *Nucl. Phys.* **B 283** (1987) 165.

[11] E. Witten, *New issues in manifolds of* SU(3) *holonomy*, *Nucl. Phys.* **B 268** (1986) 79.

[12] P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142].

[13] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[14] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[15] L. Susskind and E. Witten, *The holographic bound in anti-de Sitter space*, hep-th/9805114.

[16] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS five-branes and holography*, *J. High Energy Phys.* **10** (1998) 004 [hep-th/9808149].

[17] H. Verlinde, *Supersymmetry at large distance scales*, hep-th/0004003.

[18] C. Schmidhuber, *Micrometer gravitinos and the cosmological constant*, *Nucl. Phys.* **B 585** (2000) 385 [hep-th/0005248].

[19] K. Benakli and Y. Oz, *Small instantons and weak scale string theory*, *Phys. Lett.* **B 472** (2000) 83 [hep-th/9910090].

[20] E. Witten, *Small instantons in string theory*, *Nucl. Phys.* **B 460** (1996) 541 [hep-th/9511030].

[21] M.J. Duff and J.X. Lu, *Elementary five-brane solutions of $D = 10$ supergravity*, *Nucl. Phys.* **B 354** (1991) 141;
A. Strominger, *Heterotic solitons*, *Nucl. Phys.* **B 343** (1990) 167;
C.G. Callan Jr., J.A. Harvey and A. Strominger, *World sheet approach to heterotic instantons and solitons*, *Nucl. Phys.* **B 359** (1991) 611.

[22] M. Green and J. Schwarz, *Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117;
M. Green and J. Schwarz, *Infinity cancellations in* SO(32) *superstring theory*, *Phys. Lett.* **B 151** (1985) 21.

[23] M.J. Duff, R. Minasian and E. Witten, *Evidence for heterotic/heterotic duality*, *Nucl. Phys.* **B 465** (1996) 413 [hep-th/9601036].

[24] V.A. Rubakov and M.E. Shaposhnikov, *Extra space-time dimensions: towards a solution to the cosmological constant problem*, *Phys. Lett.* **B 125** (1993) 139.

[25] E. Verlinde and H. Verlinde, *RG-flow, gravity and the cosmological constant*, *J. High Energy Phys.* **05** (2000) 034;
C. Schmidhuber, *AdS_5 and the 4d cosmological constant*, *Nucl. Phys.* **B 580** (2000) 140 [hep-th/9912156].

[26] P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, *Vacuum configurations for superstrings*, *Nucl. Phys.* **B 258** (1985) 46.

[27] For a review and original references, see H.P. Nilles, *Gaugino condensation and supersymmetry breakdown*, *Int. J. Mod. Phys.* **A 5** (1990) 4199.

[28] P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, *Nucl. Phys.* **B 593** (2001) 99 [hep-th/0003198].

[29] P. Berglund and P. Mayr, *Heterotic string/F-theory duality from mirror symmetry*, *Adv. Theor. Math. Phys.* **2** (1999) 1307 [hep-th/9811217];
G. Rajesh, *Toric geometry and F-theory/heterotic duality in four dimensions*, *J. High Energy Phys.* **12** (1998) 018;
D.-E. Diaconescu and G. Rajesh, *Geometrical aspects of fivebranes in heterotic/F-theory duality in four dimensions*, *J. High Energy Phys.* **06** (1999) 002;
R. Donagi, B.A. Ovrut and D. Waldram, *Moduli spaces of fivebranes on elliptic Calabi-Yau threefolds*, *J. High Energy Phys.* **11** (1999) 030;
P. Berglund and P. Mayr, *Stability of vector bundles from F-theory*, *J. High Energy Phys.* **12** (1999) 009;
A. Grassi, Z. Guralnik and B. Ovrut, *Five-brane BPS states in heterotic M-theory*, hep-th/0005121.

[30] A. Klemm, W. Lerche, P. Mayr, C. Vafa and N. Warner, *Self-dual strings and $N = 2$ supersymmetric field theory*, *Nucl. Phys.* **B 477** (1996) 746 [hep-th/9604034].

[31] S. Kachru and C. Vafa, *Exact results for $N = 2$ compactifications of heterotic strings*, *Nucl. Phys.* **B 450** (1995) 69 [hep-th/9505105];
S. Ferrara, J. A. Harvey, A. Strominger and C. Vafa, *Second quantized mirror symmetry*, *Phys. Lett.* **B 361** (1995) 59 [hep-th/9505162].

[32] E. Witten, *Some comments on string dynamics*, hep-th/9507121;
M. Bershadsky, C. Vafa and V. Sadov, *D-strings on D-manifolds*, *Nucl. Phys.* **B 463**

(1996) 398 [hep-th/9510225];
A. Klemm and P. Mayr, *Strong coupling singularities and non-abelian gauge symmetries in $n = 2$ string theory*, *Nucl. Phys.* **B 469** (1996) 37 [hep-th/9601014];
S. Katz, D.R. Morrison and M.R. Plesser, *Enhanced gauge symmetry in type-II string theory*, *Nucl. Phys.* **B 477** (1996) 105 [hep-th/9601108].

[33] E. Witten, *Heterotic string conformal field theory and ADE singularities*, hep-th/9909229.

[34] P. Mayr, *Conformal field theories on K3 and three-dimensional gauge theories*, *J. High Energy Phys.* **08** (2000) 042 [hep-th/9910268].

[35] M. Rozali, *Hypermultiplet moduli space and three dimensional gauge theories*, *J. High Energy Phys.* **12** (1999) 013 [hep-th/9910238].

[36] I. Antoniadis, S. Ferrara, R. Minasian and K.S. Narain, *R^4 couplings in M- and type-II theories on Calabi-Yau spaces*, *Nucl. Phys.* **B 507** (1997) 571 [hep-th/9707013].

[37] S. Katz, A. Klemm and C. Vafa, *Geometric engineering of quantum field theories*, *Nucl. Phys.* **B 497** (1997) 173 [hep-th/9609239];
S. Katz, P. Mayr and C. Vafa, *Mirror symmetry and exact solution of 4d $N = 2$ gauge theories, 1*, *Adv. Theor. Math. Phys.* **1** (1998) 53 [hep-th/9706110].

[38] Work in progress.

[39] O.J. Ganor and A. Hanany, *Small e_8 instantons and tensionless non-critical strings*, *Nucl. Phys.* **B 474** (1996) 122 [hep-th/9602120];
N. Seiberg and E. Witten, *Comments on string dynamics in six dimensions*, *Nucl. Phys.* **B 471** (1996) 121 [hep-th/9603003].

[40] P. Ginsparg, *Comment on toroidal compactification of heterotic superstrings*, *Phys. Rev.* **D 35** (1987) 648.

[41] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [hep-th/9512059].

[42] E. Witten, *Five-branes and M-theory on an orbifold*, *Nucl. Phys.* **B 463** (1996) 383 [hep-th/9512219].

[43] M.J. Duff, J.T. Liu and R. Minasian, *Eleven-dimensional origin of string/string duality: a one-loop test*, *Nucl. Phys.* **B 452** (1995) 261 [hep-th/9506126].

[44] K. Dasgupta and S. Mukhi, *Orbifolds of M-theory*, *Nucl. Phys.* **B 465** (1996) 399 [hep-th/9512196];
A. Sen, *M-theory on $(K3 \times S^1)/\mathbb{Z}_2$*, *Phys. Rev.* **D 53** (1996) 6725 [hep-th/9602010].

[45] R. Gueven, *Black p-brane solutions of $D = 11$ supergravity theory*, *Phys. Lett.* **B 276** (1992) 49.

[46] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *The universe as a domain wall*, *Phys. Rev.* **D 59** (1999) 086001 [hep-th/9803235].

[47] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136;
K. Pilch and N.P. Warner, $N = 1$ *supersymmetric renormalization group flows from IIB supergravity*, hep-th/0006066.

[48] J.D. Blum and K. Intriligator, *New phases of string theory and 6d RG fixed points via branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 199 [hep-th/9705044];
J.D. Blum and K. Intriligator, *Consistency conditions for branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 223 [hep-th/9705030];
P.S. Aspinwall, *Point-like instantons and the* $\mathrm{Spin}(32)/\mathbb{Z}_2$ *heterotic string*, *Nucl. Phys.* **B 496** (1997) 149 [hep-th/9612108];
P.S. Aspinwall and D.R. Morrison, *Point-like instantons on K3 orbifolds*, *Nucl. Phys.* **B 503** (1997) 533 [hep-th/9705104];
P. Berglund and P. Mayr, *Heterotic string/F-theory duality from mirror symmetry*, *Adv. Theor. Math. Phys.* **2** (1999) 1307 [hep-th/9811217];
P.S. Aspinwall, *Aspects of the hypermultiplet moduli space in string duality*, *J. High Energy Phys.* **04** (1998) 019 [hep-th/9802194];
P.S. Aspinwall and R.Y. Donagi, *The heterotic string, the tangent bundle and derived categories*, *Adv. Theor. Math. Phys.* **2** (1998) 1041 [hep-th/9806094].

[49] M. Bershadsky, C. Vafa and V. Sadov, *D-branes and topological field theories*, *Nucl. Phys.* **B 463** (1996) 420 [hep-th/9511222].

[50] S. Kachru, N. Seiberg and E. Silverstein, *SUSY gauge dynamics and singularities of 4d* $N = 1$ *string vacua*, *Nucl. Phys.* **B 480** (1996) 170 [hep-th/9605036].